Spin Squeezing in Degenerate and Non-degenerate Atomic Vapors: Some Approaches to Atomic Entanglement

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In this paper we discuss our recent work on the creation of spin-squeezed states in both thermal (room temperature) gasses and in spinor Bose-Einstein condensates.

Key words: Squeezing; Entanglement; BEC; Spin.

Entanglement is one of the most intrinsically *quantum* aspects of the quantum theory; Entanglement is of importance to advanced quantum technologies such as quantum computers and teleportation devices. Entanglement of *many* atoms has developed into an important frontier of contemporary physics and quantum optics. In this paper we describe our recent work on the creation of entanglement via the creation of spin-squeezed states of a gas of neutral atoms. The specific problem of spin-squeezing is a particularly interesting aspect of entanglement because it can have important consequences for precision measurements and measurements where quantum "noise" is important.

Consider the case of a neutral atom atomic clock: many atom entanglement can, in principle, dramatically improve the measurement sensitivity of a typical clock (or any atom interferometer) from $1/\sqrt{N}$ to $1/N$, where N is the number of detected atoms. In turn, this improvement implies a much greater short-term stability for the same number of detected atoms. Quite recently, a fundamental performance barrier for the cold-atom "fountain" clocks has been observed: the so-called "projection-noise" limit [1]. The entanglement of the many atoms used in such a clock measurement promises to provide an intriguing quantum means to break this limit.

The extensive research activity devoted to the generation of the non-classical states of the electromagnetic field and to their applications to reduce the uncertainty of phase measurement is therefore now being paralleled by the quest for similar states in other physical systems, systems which could be used to improve measurements relevant to a broader range of physical effects. A notable example of the latter is represented by the spin squeezed state (SSS) of atomic particles [2, 3]. Just as squeezed states of light allow for a reduction in the measurement uncertainty due to quantum fluctuations of light, SSS of atoms and ions hold the promise to reduce uncertainties induced by quantum fluctuations in atomic measurement systems.

As a starting point, we consider the measurement of atomic projection noise. In a variety of experiments, quantum spin-noise has already been investigated [4]. For this paper we focus on recent work carried out in our laboratories in which we have demonstrated the ability to detect atomic spin-noise at the shot-noise limit [5]. We then describe how we used our apparatus to explore spin state measurement and control, and how we have used it to realize non-classical states of the *collective* atomic spin of the gas.

1. Spin Squeezing of a Room Temperature Atomic Vapor in a Simple Glass Cell

1.1. Measuring the Spin-state and Spin Noise using Paramagnetic Faraday Rotation

Our techniques are based on a direct measurement of the rotation of the polarization axis of a nearly-

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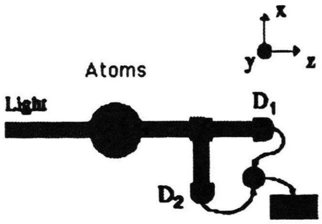

Fig. 1. Experimental set-up for measurement of the paramagnetic Faraday rotation and of the collective atomic spin. A linearly polarized laser field interacts with an atomic sample and the rotation of the plane of polarization is detected using a polarization sensitive beam splitter and two detectors (D_1 and D_2)

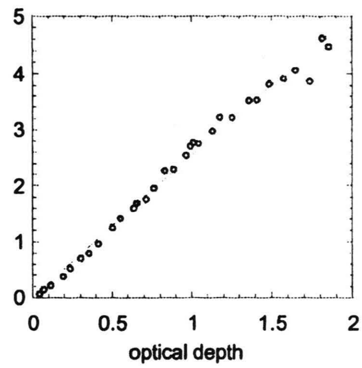

Fig. 2. Measured spin noise as a function of optical thickness of the sample. The linear dependence of the noise on optical depth, and hence atom number, is a signature of atomic shot-noise [5].

resonant linearly polarized laser field passed though an atomic [5,6] sample using a polarimeter as shown in Figure 1.

In this system, a collimated laser beam, linearly polarized at a 45° angle between the x and y axes is passed through an atomic sample. For atoms with ground state angular momentum of $\hbar/2$, in an off resonant atom-photon interaction, the interaction Hamiltonian scales as $s_z(t)\boldsymbol{F}_z(t)$, where $\boldsymbol{F}$ is the z-projection of the collective atomic spin ($\boldsymbol{F}_z(t) = \Sigma_j \boldsymbol{F}_x^j$, where $\boldsymbol{F}_z^j$ is an individual atomic spin) and where s_z is the photon spin operator defined as: $s_z = 1/2[a_\mathrm{v}^\dagger(t)a_\mathrm{h}(t) + a_\mathrm{h}^\dagger(t)a_\mathrm{v}(t)]$ (here the a's are creation and annihilation operators for horizontally and vertically polarized probe field modes). The physical effect is that the polarization state of the forward scattered light is rotated by an amount which depends on the value of the collective atomic spin $\boldsymbol{F}_z(t)$. For our work, what is key is that the noise on the rotation angle is influenced by the atomic (quantum) spin-noise. In this polarimeter, after the light passes through the sample, it is projected onto the vertical and horizontal axes by a polarization sensitive beam-splitter cube (a Glan-Thompson beam splitter) and detected by photodetectors D_1 and D_2. It is the signal generated by the two detectors which is used to determine the orientation of the collective atomic spin, and, for our work, to determine the atomic spin noise.

We stress that in this overview paper we are interested in spin-fluctuations and hence we focus primarily on the "spin-noise" in the Faraday rotation measurement. That is, we focus on the fluctuations in the rotation angle. Using our polarimeter, we measure these fluctuations in two ways. In the first mode, probe pulses of duration 2 - 300 ns are passed through the atomic sample. As the pulses arrive at detectors D_1 and D_2 the resulting signals are digitized and recorded by a computer. Every 10,000 or so pulses, the difference signal (D_1-D_2) and the variances are calculated. In a second measure scheme, a continuous (CW) laser beam is passed through the sample. To examine the noise, the difference current from the detectors, again (D_1-D_2), is found using a balanced difference amplifier, and the noise spectrum is measured using a high-performance spectrum analyzer.

In our first set of experiments [5] we demonstrated that we could isolate the noise contribution in the polarimeter due to the atomic shot-noise. In this work, the measurements were made using an unpolarized (thermal) atomic sample. Measured as a function of the optical thickness of the sample (see Fig. 2), we observed a spin-noise contribution which scaled linearly with the number of atoms in the interaction zone (defined by the profile of the probe laser beam). This provided a clear signature of atomic "spin-shot-noise".

1.2. Spin-Squeezing

It is now widely recognized that the use of squeezed atomic states has the potential for substantial improvement in the sensitivity of atom interferometers such as the two-zone Ramsey atomic clock and the atomic fountain clock. This realization has led to diverse theoretical and experimental work on schemes to realize atomic spin squeezing. Building upon the seminal work on photon number squeezing by Kitagawa and Yamamoto [7], Kitagawa and Ueda [2] proposed and clarified the basic issues concerning the definition and preparation of spin squeezed states (SSS's). This was

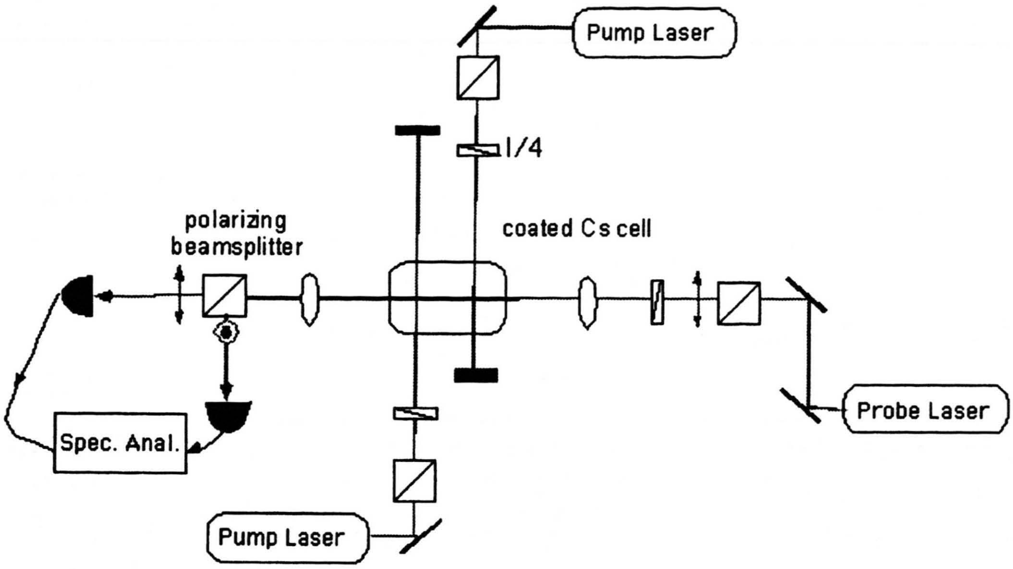

Fig. 3. Modified Polarimeter Apparatus. Two additional lasers are used to pump the sample into a CSS. In this measurement the spin noise was examined using a CW probe laser and a low-frequency spectrum analyzer.

followed by studies of the production of squeezed atomic states [2,3], the transfer of squeezing from incident squeezed light to an assembly of excited state cold atoms, and measurements related to a collection of squeezed atoms [4 - 6, 8 - 11].

In a recent development, the realization of spin-squeezing of ground state spins in a vapor of cesium atoms has been demonstrated in our laboratories [11]. This spin-squeezing can be understood as a particular case of quantum entanglement in which the correlations between spins are introduced in such a way that the effect is to reduce fluctuations associated with projection noise in the measurement of the collective atomic spin of the vapor. We prepared the spin-squeezed sample by first state-selecting the atomic vapor to prepare a completely polarized sample in a coherent spin-state (CSS) which represents a minimum (spin) uncertainty spin state of the sample [12]. The CSS is a state in which atoms of spin F are completely in the state with magnetic projection $m_F = F$ (i. e. $|\Psi\rangle = |F = F, m_F = F\rangle$). The state was then "spin-squeezed" using a so-called QND-type interaction provided by the laser-atom interaction [10]. Under these conditions the collective spin state of the system was caused to undergo a slow oscillation around the polarization axis (the axis of the collective spin), and the noise in the measurement of the rotation angle was evaluated. We note that our choice to examine the noise in the rotation angle measurement is not at all restrictive, as the measurement is the direct analogue of an atomic-clock type experiment such as performed in a cold-atom fountain or in an EDM experiment (an experiment which searches for the permanent dipole moment of the electron).

To create the CSS, we optically pumped the atomic sample using two additional diode lasers. The atomic sample was a cesium vapor (saturated vapor pressure at 300K) contained in a glass cell whose inner walls were coated with high atomic-weight paraffin. This coating minimized the atomic spin diffusion at the walls, permitting long spin lifetimes ($\sim$ minute) and high ground state polarization ($\sim 95\%$). The modified polarimeter apparatus is shown in Figure 3.

In Fig. 4 we show the noise spectrum generated from the difference current of the two photo-detectors shown in the polarimeter set-up of Figure 3. The dotted line is the atomic shot-noise level that defines the "standard quantum limit" for the measurement. The sharp peak at 16 MHz is due to the RF field used to cause the spin oscillations. Note that the noise floor is well below the ("unsqueezed") projection-noise level of the sample.

These data clearly shown a 75% spin-noise reduction below the SQL, which, for a polarization of 95%, corresponds to 70% spin-squeezing [11].

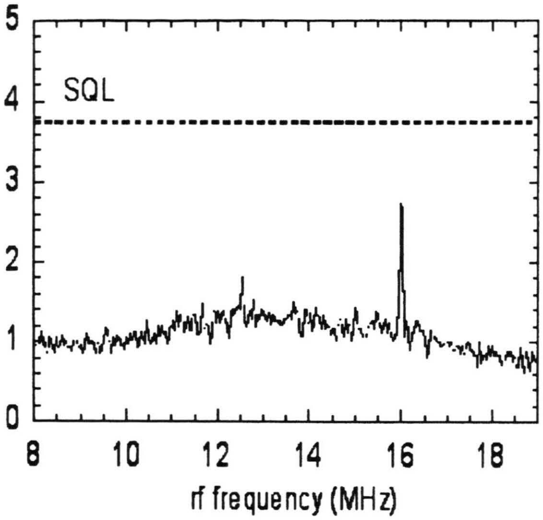

Fig. 4. Noise Spectrum of the SSS sample. The dashed line shows the standard quantum limit (SQL) defined by the atomic shot-noise level of the unsqueezed sample. The vertical scale is in arbitrary noise (variance) units. The sharp peak is the RF signal used to cause the spin oscillation about the polarization axis measured in our spin-interferometer (see text) [11].

2. Spin Squeezing of a Spinor BEC

Until recently, work on spin-squeezing has been limited to the quantum state control of non-degenerate systems. Motivated by the experimental work on ^{23}Na by the group at MIT, there has been intensive work on spin manipulation in the context of spinor BECs. In our group, theoretical analysis [13] addressed the issue of collective spin properties in spinor BECs, and we showed that the spinor ground state can be highly entangled [14]. Recently, Sørensen *et al.* [15] studied the possibility of realizing spin squeezing in a spinor condensate by taking advantage of the internal nonlinear atom-atom interaction in the condensate. In our group, we also realized the potential for obtaining spin-squeezing in a condensate and developed a quantum control technique which allows one to prepare *arbitrary* Dicke spin states [16], and, most particularly, a maximally entangled state, by means of applying properly time-sequenced external RF fields which couple the spin states of the condensate. We consider a two-component condensate with N_i atoms in component i, and $N_1 + \mathrm{N}_2 = N$. A general state of the system can be written as a superposition of number difference states.

Alternatively, we introduce the effective angular momentum quantum number

$$|\Psi\rangle = |N_1, N_2\rangle$$

$j = (N_1+N_2)/2 = N/2$ and the z-projection quantum number $m = (N_2 - N_1)/2$ and reexpress the state in terms of angular momentum states. In particular, for the the CSS

$$|j, -j\rangle = |N_1 = N, N_2 = 0\rangle .$$

Our approach allows one to construct arbitrary states of the system from this ground state.

In a key paper, Kitagawa and Yamamoto [7] showed that number-squeezing of the electromagnetic field could be realized by making use of a Kerr nonlinear medium. The high-performance squeezing resulted from the fact that the Hamiltonian giving rise to squeezing involves a quartic four-wave process, whereas ordinary squeezing is a quadratic process. Subsequently, Kitagawa and Ueda [2] showed that it was possible, analogously, to produce squeezed spin states by making use of spin Hamiltonians quadratic in the spin operators. Our work relies on the existence of exactly such a term in the Hamiltonian describing the dynamics of a coupled two-component condensate. Briefly, beginning with a two-mode Hamiltonian $\mathcal{H}$ for the two-component condensate [16], we find that we can write $\mathcal{H} = \kappa \boldsymbol{J}_z^2 + \Omega_x \boldsymbol{J}_x + \Omega_y \boldsymbol{J}_y$. Here, we introduce an effective nonlinear coupling coefficient κ which is a weighted average of the sum of the two self-interaction scattering lengths (a_{11} and a_{22}) and the cross-species scattering length a_{12}. Also, the Ωs are the Rabi coupling frequencies between the two spin levels which together form the two-component condensate. With such a Hamiltonian, procedures for achieve spin squeezing in the equatorial plane of a non-degenerate system were considered by Kitagawa and Ueda [2], and more recently by Law *et al.* [17].

In one such scheme, a $\pi/2$ rf pulse along the y-axis is applied to the ground state at $t = 0$. The pulse is assumed to be so short that the effects of the nonlinear interaction during the pulse are negligible. The effect is to align the spin vector along the negative x-axis. After the pulse, the system evolves under the Hamiltonian $\mathcal{H} = \kappa \boldsymbol{J}_z^2$ until a second $\pi/2$ pulse is applied along x.

The difficulty with the recently considered schemes for generating an SSS of a BEC is that they do not

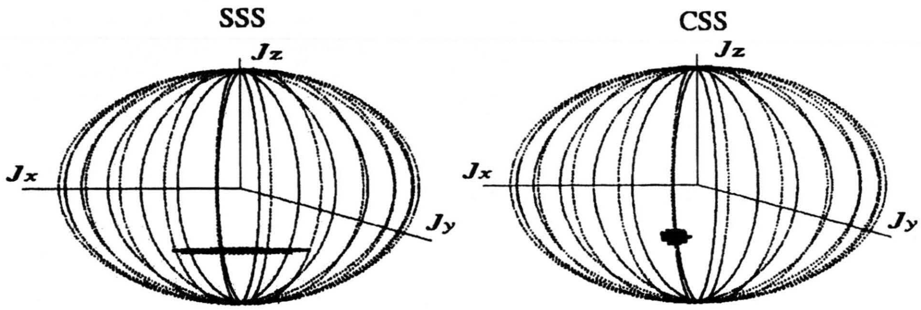

Fig. 5. Quasi-probability distribution functions for two-component BECs used in our scheme for creating sipn-squeezed condensate states [16].

allow for the creation of a squeezed state with any prescribed $\langle J_z \rangle = m_0 \neq 0$. Using our procedure however, this need not to be the case. We use the following method: First, a coupling pulse of appropriate strength is applied to the ground state $|j, -j\rangle$ creating an initial CSS with $\langle J_z \rangle = m_0$. The pulse is along the x-axis. Following this pulse, we apply a coupling field along the negative x-axis. During the subsequent time evolution, both $\langle J_z \rangle$ and $\langle J_z^2 \rangle$ start to oscillate. At specific times, when $\langle J_z \rangle$ returns back to its initial value of m_0, $\langle J_z^2 \rangle$ comes close to a local minimum which is less than the initial variance. Provided the coupling field is turned off at these precise times, a state squeezed along $\boldsymbol{J}_z$ with $\langle J_z \rangle = m_0$ is created. For large enough squeezing, the state thus prepared can be regarded as an approximation of the Dicke state $|j, m_0\rangle$. This state is the spin space analog of the Fock state. In Fig. 5 we show the quasi-probability distribution functions for an example CSS and the resulting SSS created using our scheme.

Highly entangled states of massive particles are of great importance in fundamental physics and are at the heart of many important "puzzles and paradoxes" as well as being important for applications in quantum information and quantum measurement. A great deal of effort has been directed toward the creation of entangled states of atoms and two-, three- and four-particle [18] entanglement has been successfully demonstrated experimentally in trapped ions, Rydberg atoms, and using cavity QED. It is well recognized that a further increase of the number of entangled particles in these systems presents a severe experimental challenge. We believe that spin-squeezed atomic vapors and the two-component, atomic BECs offer exceptionally powerful systems in which to generate and study entanglement on a macroscopic level.

The work described in this paper is the work of two separate collaborations. The non-degenerate spin-squeezing work was carried out in collaboration with Dr. Alex Kuzmich and Professor Leonard Mandel. The work on Dicke states of a BEC was carried out in collaboration with Dr. H. Pu, Dr. S. Raghavan and Professor P. Meystre. This work was supported by the National Science Foundation and the Office of Naval Research.

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